

Cherry Hinton Primary School Calculation Policy



Objectives

- To demonstrate how we teach different forms of calculation at Cherry Hinton Primary School.
- To demonstrate what methods are introduced in each year group, although some children may continue to use methods from earlier year groups to support their understanding.
- To develop a deep understanding of the four operations (addition, subtraction, multiplication and division).
- To prepare children for using formal methods fluently for all four operations, as is required by year 6 end of year tests.
- To help teachers and staff members ensure that calculation is taught consistently across the school.
- To aid teachers in helping children who may need extra support.
- To help parents, carers and other family members support children's learning by letting them know the expectations for their child's year group and by providing an explanation of the methods used in school.

Methods

The **CPA (concrete, pictorial, abstract)** approach is used in maths lessons throughout the school to develop children's understanding.

Methods are first introduced using **concrete** (physical) resources, giving children a chance to solve a problem by physically 'doing' it. For example, children could use real oranges to solve a problem about oranges. They progress from there to using physical resources such as counters, cubes or base 10 blocks.

Next is the **pictorial** stage, where children use pictures linked to the previous 'doing' stage to 'see' the problem. For example, they may draw the counters they have previously worked with.

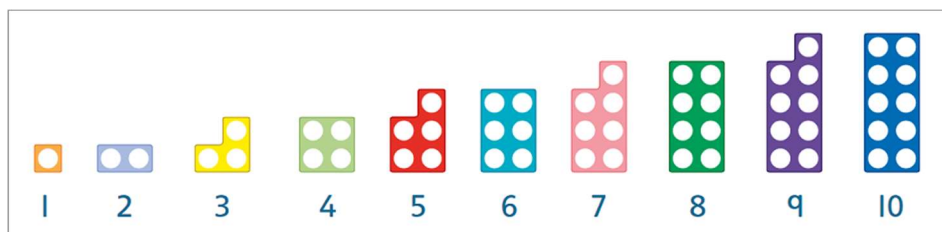
Finally is the **abstract**, symbolic stage, where children use mathematical symbols or numbers to model a problem. Children will not progress to this stage until they have understood what is going on behind these symbols using the concrete and pictorial methods.

Resources

We use a range of concrete (physical) resources to teach maths in our school. These are used to deepen children's understanding of number and place value, and support them in calculations before they move on to formal, written methods. Examples of how these are used can be seen in the breakdowns later in the policy.

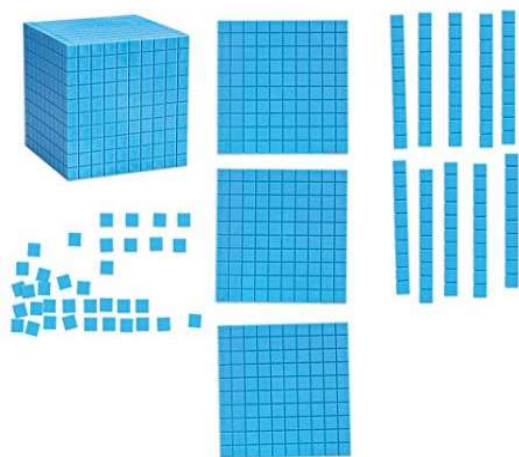
Numicon

There are Numicon pieces to represent the numbers 1 to 10, to give children an idea of what these numbers 'look like'.



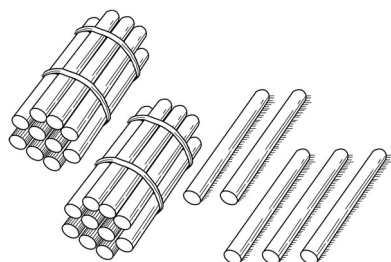
Base 10 Blocks/ Dienes

Base 10 blocks are used for children to understand place value and how ones, tens, hundreds, thousands, etc. link to one another.



Bundles of Straws

These can be created at home as an alternative to base 10 blocks. Individual straws and bundles of ten and one hundred can represent ones, tens and hundreds.



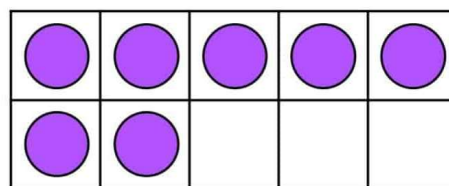
Place Value Counters

Place Value counters are used higher up the school to represent the place value of the digits in numbers e.g. hundreds, tens, ones.



Ten Frames

A ten frame is a 5 by 2 grid that can be filled with counters or dots. It helps children to visualise numbers and their relationship to 5 or 10. Multiple ten frames could be used to represent multiples of 10.



Part 1: Number Facts and Mental Methods

(based on DfE 'Ready to Progress' criteria)

Fluency with number facts is incredibly important in helping children to become confident and efficient with written calculations. Fluency means either knowing a fact off by heart, or being able to rapidly generate it (in 2-3 seconds) using a mental strategy.

Below, the facts and strategies that children learn in each year group are laid out. These are taught in school in maths lessons, as well as through 'Number Talks', where different mental maths questions are posed to the children and different strategies are discussed and demonstrated visually. Children can be supported to learn these key facts and strategies through regular practise and discussing strategies aloud e.g. "I know $5 + 5 = 10$, and $5 + 6$ is one more, so it's 11." For some calculations, there may be more than one efficient strategy that children may choose to use.

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition and Subtraction	Fluency with addition and subtraction facts to 10	Fluency with addition and subtraction facts to 20	Consolidation of addition and subtraction facts to 20. Scaling facts by 10.	Scaling facts by 100.	Scaling facts by 0.1 or 0.001.	Consolidation
Multiplication and Division		Developing 2, 5 and 10 times tables and division facts.	Fluency with 2, 5, 10, 3, 4 and 8 times tables and division facts.	Fluency with all times tables and division facts to 12×12 .	Consolidation of times tables and division acts to 12×12 .	Consolidation

Addition and Subtraction

Facts to 20 – taught in year 1 and 2, consolidated in year 3.

In Key Stage 1, children should become fluent in addition and subtraction facts to 20. These are displayed in the grids below. The facts children should become fluent with in year 1 are in white, and the facts that children should become fluent with in year 2 are in blue. Children learn a range of strategies to support fluency, including:

- Reordering numbers when adding (e.g. $1+5 = 5+1$)
- Finding one more and one less (for $+1$ and -1)
- Counting in odds and evens (for $+2$ and -2)
- Number bonds to 10 (e.g. $3+7=10$)
- Knowing what zero does (for $+0$ and -0)
- Learning doubles (e.g. $3+3=6$)
- Using near doubles (e.g. $3+4 \rightarrow 3+3=6, 6+1=7$)
- Finding ten more and ten less (for $+10$ and -10)
- Bridging through ten (e.g. $8+5 \rightarrow 8+2=10, 10+3=13$)
- Adjusting from known facts (e.g. $6+9 \rightarrow 6+10=16, 16-1=15$)

The basis of many of these strategies is a strong understanding of numbers to 20. Children will have opportunities to visualise these numbers represented in different ways, including through Numicon and ten frames, and to explore how these can be broken apart.

+	0	1	2	3	4	5	6	7	8	9	10
0	0+0	0+1	0+2	0+3	0+4	0+5	0+6	0+7	0+8	0+9	0+10
1	1+0	1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9	1+10
2	2+0	2+1	2+2	2+3	2+4	2+5	2+6	2+7	2+8	2+9	2+10
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5	5+0	5+1	5+2	5+3	5+4	5+5	5+6	5+7	5+8	5+9	5+10
6	6+0	6+1	6+2	6+3	6+4	6+5	6+6	6+7	6+8	6+9	6+10
7	7+0	7+1	7+2	7+3	7+4	7+5	7+6	7+7	7+8	7+9	7+10
8	8+0	8+1	8+2	8+3	8+4	8+5	8+6	8+7	8+8	8+9	8+10
9	9+0	9+1	9+2	9+3	9+4	9+5	9+6	9+7	9+8	9+9	9+10
10	10+0	10+1	10+2	10+3	10+4	10+5	10+6	10+7	10+8	10+9	10+10

-	0	1	2	3	4	5	6	7	8	9	10
0	0 - 0										
1	1 - 0	1 - 1									
2	2 - 0	2 - 1	2 - 2								
3	3 - 0	3 - 1	3 - 2	3 - 3							
4	4 - 0	4 - 1	4 - 2	4 - 3	4 - 4						
5	5 - 0	5 - 1	5 - 2	5 - 3	5 - 4	5 - 5					
6	6 - 0	6 - 1	6 - 2	6 - 3	6 - 4	6 - 5	6 - 6				
7	7 - 0	7 - 1	7 - 2	7 - 3	7 - 4	7 - 5	7 - 6	7 - 7			
8	8 - 0	8 - 1	8 - 2	8 - 3	8 - 4	8 - 5	8 - 6	8 - 7	8 - 8		
9	9 - 0	9 - 1	9 - 2	9 - 3	9 - 4	9 - 5	9 - 6	9 - 7	9 - 8	9 - 9	
10	10 - 0	10 - 1	10 - 2	10 - 3	10 - 4	10 - 5	10 - 6	10 - 7	10 - 8	10 - 9	10 - 10
11		11 - 1	11 - 2	11 - 3	11 - 4	11 - 5	11 - 6	11 - 7	11 - 8	11 - 9	11 - 10
12			12 - 2	12 - 3	12 - 4	12 - 5	12 - 6	12 - 7	12 - 8	12 - 9	12 - 10
13				13 - 3	13 - 4	13 - 5	13 - 6	13 - 7	13 - 8	13 - 9	13 - 10
14					14 - 4	14 - 5	14 - 6	14 - 7	14 - 8	14 - 9	14 - 10
15						15 - 5	15 - 6	15 - 7	15 - 8	15 - 9	15 - 10
16							16 - 6	16 - 7	16 - 8	16 - 9	16 - 10
17								17 - 7	17 - 8	17 - 9	17 - 10
18									18 - 8	18 - 9	18 - 10
19										19 - 9	19 - 10
20											20 - 10

Scaling Facts in KS2

In key stage 2, children will use the facts they are fluent with and scale them to apply them to new contexts.

In year 3, they will scale by 10 – if they know $3 + 4 = 7$, they'll use this to quickly calculate $30 + 40 = 70$. In

year 4, they scale by 100 → $300 + 400 = 700$. In year 5, they scale by 0.1 or 0.01 → $0.3 + 0.4 = 0.7$ and $0.03 + 0.04 = 0.07$.

Multiplication and Division

2, 5 and 10 times tables – introduced in year 2 and consolidated in year 3.

Children are expected to become fluent in the 2, 5 and 10 times tables and corresponding division facts by the end of year 3. Some facts may be learned by heart, others may be generated by strategies including:

- Counting or chanting multiples of 2, 5 and 10.
- Counting on from a known multiple (e.g. I know $5 \times 5 = 25$, so can add 5 more to find 5×6)
- Spotting patterns (e.g. multiples of 5 and end in 0 or 5)
- Reversing the numbers in a multiplication fact (e.g. $3 \times 5 = 5 \times 3$)
- Doubling a 5x table to find a 10x table, or halving a 10x table to find a 5x table.
- Using the inverse for division facts (e.g. $5 \times 3 = 15$ so $15 \div 3 = 5$)

3, 4 and 8 times tables – year 3

Children are expected to become fluent in the 4 and 8 times tables and corresponding division facts by the end of year 3. Some facts may be learned by heart, others may be generated by strategies including:

- Counting or chanting multiples of 4 and 8.
- Counting on from a known multiple (e.g. I know $8 \times 5 = 40$, so can add 8 more to find 8×6)
- Spotting patterns (e.g. multiples of 4 are always even)
- Reversing the numbers in a multiplication fact (e.g. $5 \times 8 = 8 \times 5$)
- Doubling a 2x table to find a 4x table, and doubling it again to find an 8x table.
- Using the inverse for division facts (e.g. $5 \times 8 = 40$ so $40 \div 8 = 5$)

6, 7, 9, 11 and 12 times tables – year 4

Children are expected to become fluent in all times tables and corresponding division facts up to 12×12 by the end of year 4. Some facts may be learned by heart, others may be generated by strategies including:

- Counting or chanting multiples of the number.
- Counting on from a known multiple (e.g. I know $9 \times 5 = 45$, so can add 9 more to find 9×6)
- Spotting patterns
- Reversing the numbers in a multiplication fact (e.g. $5 \times 9 = 9 \times 5$)
- Doubling a 3x table to find a 6x table, and doubling it again to find a 12x table.
- Tricks (e.g. finger trick for 9x table)
- Using the inverse for division facts (e.g. $5 \times 9 = 45$ so $45 \div 9 = 5$)

Part 2: Written Calculation Strategies

(adapted from White Rose Maths Calculation Policy)

Below is an overview of the strategies taught in each year group. The document then provides a detailed breakdown of how each operation is taught in each year group.

	EYFS/Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on- using cubes. Regrouping to make 10 using ten frame.	Adding three single digits. Use of base 10 to combine two numbers.	Column method- regrouping. Using place value counters (up to 3 digits).	Column method- regrouping. (up to 4 digits)	Column method- regrouping. Use of place value counters for adding decimals.	Column method- regrouping. Abstract methods. Place value counters to be used for adding decimal numbers.
Subtraction	Taking away ones Counting back Find the difference Part whole model Make 10 using the ten frame	Counting back Find the difference Part whole model Make 10 Use of base 10	Column method with regrouping. (up to 3 digits using place value counters)	Column method with regrouping. (up to 4 digits)	Column method with regrouping. Abstract for whole numbers. Start with place value counters for decimals- with the same amount of decimal places.	Column method with regrouping. Abstract methods. Place value counters for decimals- with different amounts of decimal places.
Multiplication	Recognising and making equal groups. Doubling Counting in multiples Use cubes, Numicon and other objects in the classroom	Arrays- showing commutative multiplication	Arrays $2d \times 1d$ using base 10	Column multiplication- introduced with place value counters. (2 and 3 digit multiplied by 1 digit)	Column multiplication Abstract only but might need a repeat of year 4 first (up to 4 digit numbers multiplied by 1 or 2 digits)	Column multiplication Abstract methods (multi-digit up to 4 digits by a 2 digit number)
Division	Sharing objects into groups Division as grouping e.g. I have 12 sweets and put them in groups of 3, how many groups? Use cubes and draw round 3 cubes at a time.	Division as grouping Division within arrays- linking to multiplication Repeated subtraction	Division with a remainder- using lollipop sticks, times tables facts and repeated subtraction. $2d$ divided by $1d$ using base 10 or place value counters	Division with a remainder Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division (up to 4 digits by a 1 digit number including remainders)	Short division Long division with place value counters (up to 4 digits by a 2 digit number) Children should exchange into the tenths and hundredths column too

Addition

Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'.

Year 1

Combining two parts to make a whole: part whole model.

Starting at the bigger number and counting on- using cubes.

Regrouping to make 10 using ten frame.

Year 2

Adding three single digits.

Use of base 10 to combine two numbers.

Year 3

Column method- regrouping: using place value counters (up to 3 digits).

Year 4

Developing year 3 strategy: column method- regrouping (up to 4 digits).

Year 5

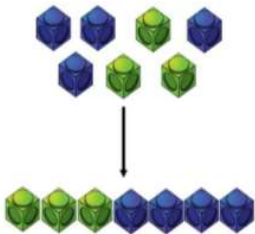
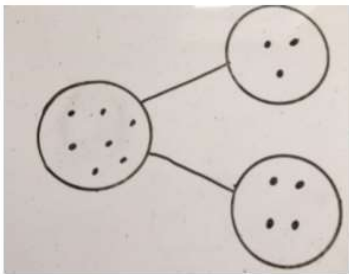
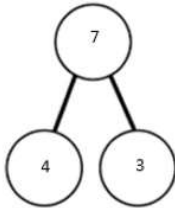
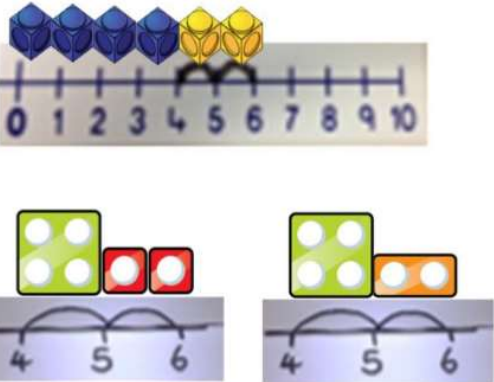
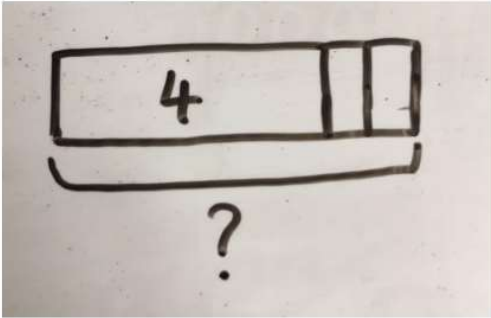
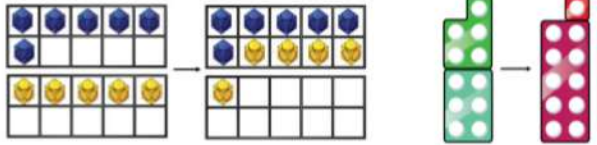
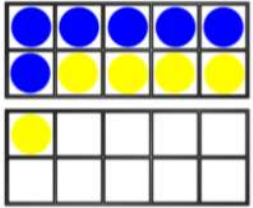
Developing year 3 strategy: Column method- regrouping (more than 4 digits).

Use of place value counters for adding decimals.

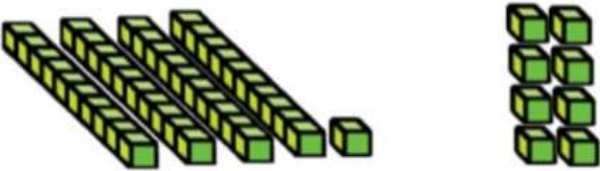
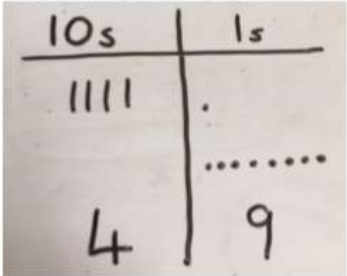
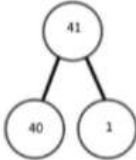
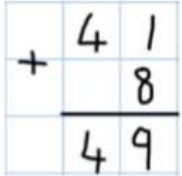
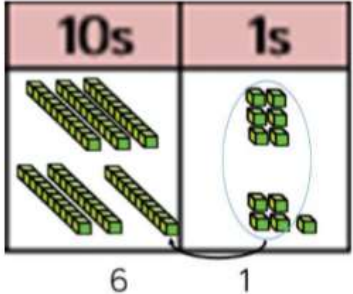
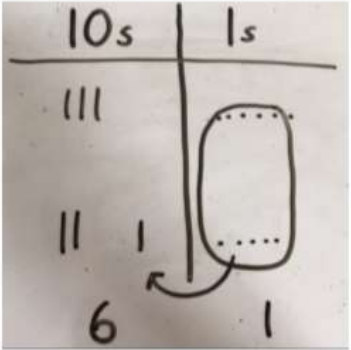
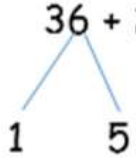
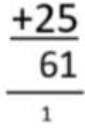
Year 6

Developing year 3 strategy: Column method- regrouping (more than 4 digits).

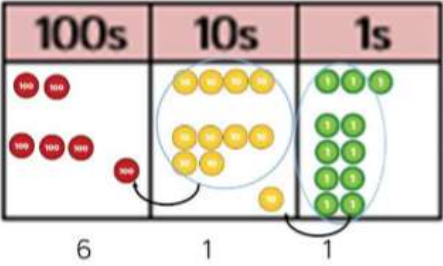
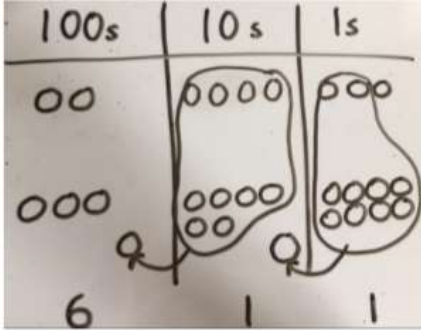
Developing year 5 strategy: Place value counters to be used for adding decimal numbers.

Concrete	Pictorial	Abstract
Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars). 	Children to represent the cubes using dots or crosses. They could put each part on a part whole model too. 	$4 + 3 = 7$ Four is a part, 3 is a part and the whole is seven. 
Counting on using number lines using cubes or Numicon. 	A bar model which encourages the children to count on, rather than count all. 	The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? $4 + 2$ 
Regrouping to make 10; using ten frames and counters/cubes or using Numicon. $6 + 5$ 	Children to draw the ten frame and counters/cubes. 	Children to develop an understanding of equality e.g. $6 + \square = 11$ $6 + 5 = 5 + \square$ $6 + 5 = \square + 4$

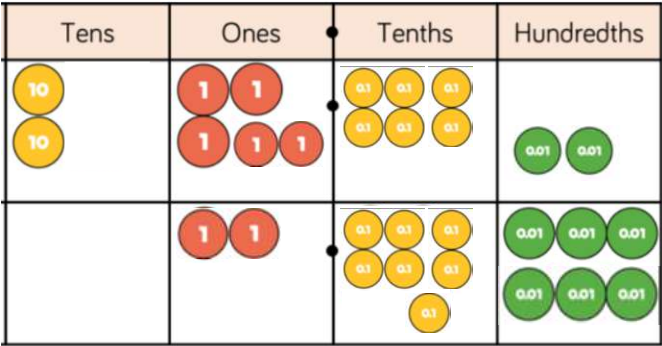
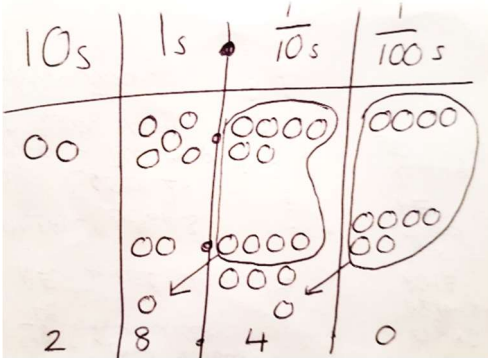
Introduced in Year 2

Concrete	Pictorial	Abstract
TO + O using base 10. Continue to develop understanding of partitioning and place value. $41 + 8$ 	Children to represent the base 10 e.g. lines for tens and dot/crosses for ones. 	$41 + 8$  $1 + 8 = 9$ $40 + 9 = 49$ 
TO + TO using base 10. Continue to develop understanding of partitioning and place value. $36 + 25$ 	Children to represent the base 10 in a place value chart. 	Looking for ways to make 10. $36 + 25 =$  $30 + 20 = 50$ $5 + 5 = 10$ $50 + 10 + 1 = 61$ Formal method: 

Introduced in Year 3; consolidated in Year 4

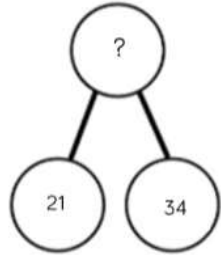
Concrete	Pictorial	Abstract
Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred. 	Children to represent the counters in a place value chart, circling when they make an exchange. 	$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ 1 \quad 1 \end{array}$

Introduced in Year 5; consolidated in Year 6

Concrete	Pictorial	Abstract
Use of place value counters to add decimals. 	Children to represent the counters in a place value chart, circling when they make an exchange. 	$\begin{array}{r} 25.64 \\ + 2.76 \\ \hline 28.40 \\ 1 \quad 1 \end{array}$

Across year groups – different presentation of problems.

Conceptual variation; different ways to ask children to solve $21 + 34$



?	
21	34

Word problems:

In year 3, there are 21 children and in year 4, there are 34 children. How many children in total?

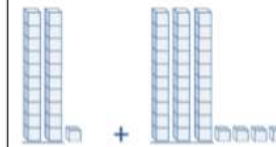
$21 + 34 = 55$. Prove it

$$\begin{array}{r} 21 \\ +34 \\ \hline \end{array}$$

$21 + 34 =$

$$\boxed{} = 21 + 34$$

Calculate the sum of twenty-one and thirty-four.



Missing digit problems:

10s	1s
	?
?	5

Subtraction

Key language: take away, less than, the difference, subtract, minus, fewer, decrease.

Year 1

Taking away ones

Counting back

Find the difference

Make 10 using the ten frame

Part whole model

Year 2

Developing year 1 strategies: counting back, find the difference, make 10, part whole model

Use of base 10

Year 3

Column method with regrouping (up to 3 digits using place value counters).

Year 4

Developing year 3 strategy: Column method with regrouping (up to 4 digits).

Year 5

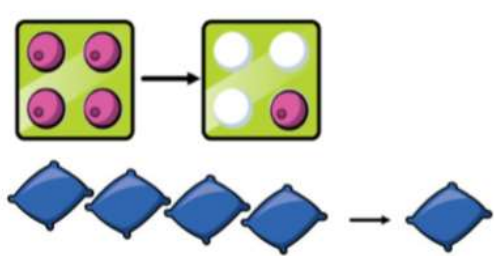
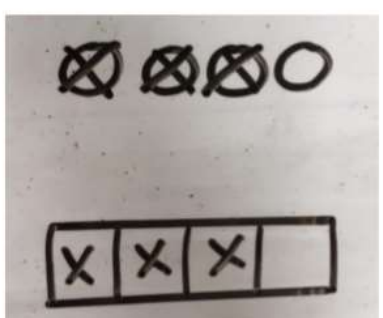
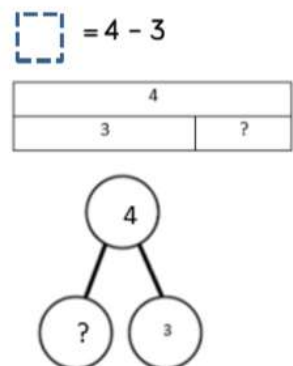
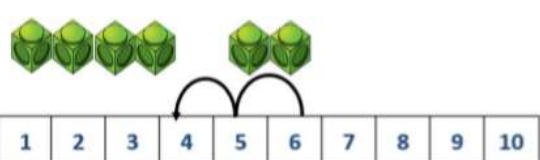
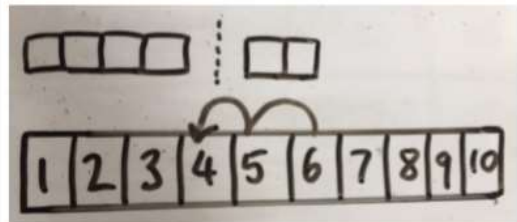
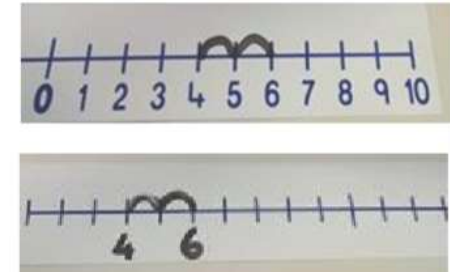
Developing year 3 strategy: Column method with regrouping - abstract for whole numbers.

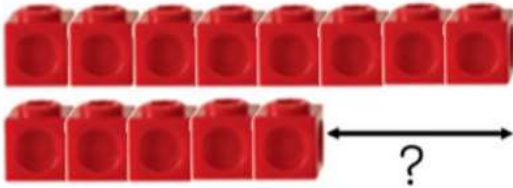
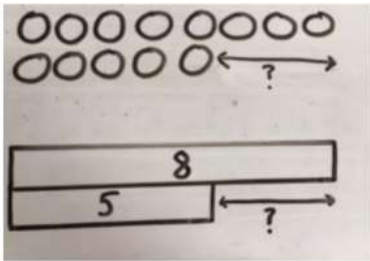
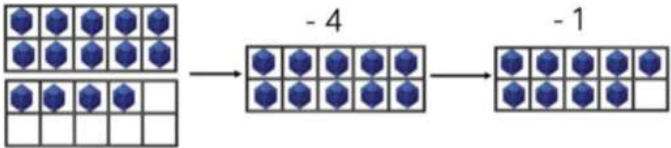
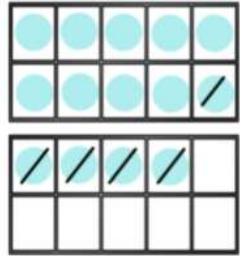
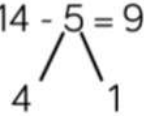
Start with place value counters for decimals- with the same amount of decimal places.

Year 6

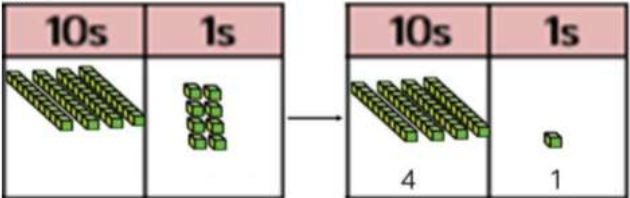
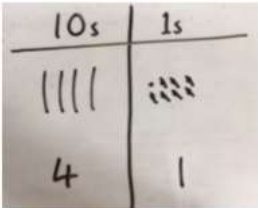
Developing year 3 strategy: Column method with regrouping - abstract methods.

Developing year 5 strategy: Place value counters for decimals- with different amounts of decimal places.

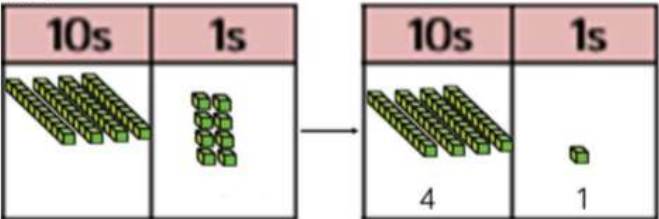
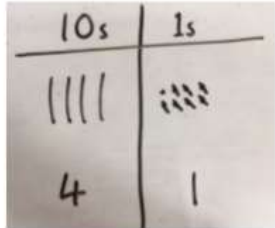
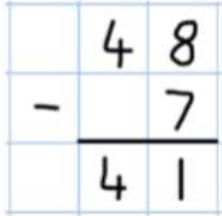
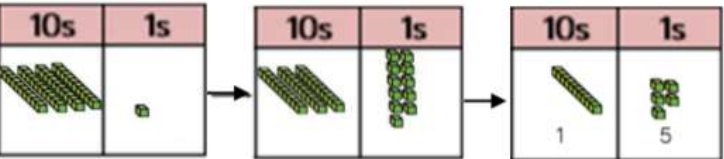
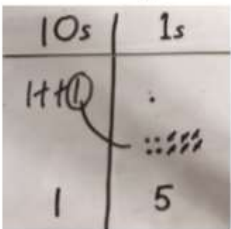
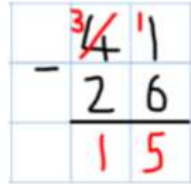
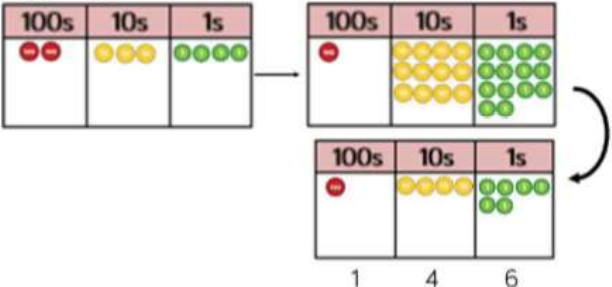
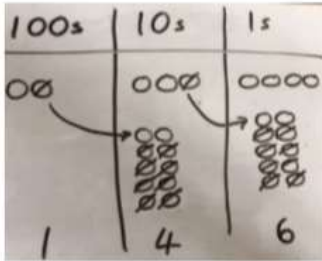
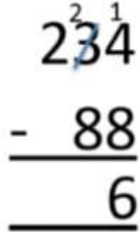
Concrete	Pictorial	Abstract
Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used). $4 - 3 = 1$ 	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used. 	$4 - 3 =$ 
Counting back (using number lines or number tracks) children start with 6 and count back 2. $6 - 2 = 4$ 	Children to represent what they see pictorially e.g. 	Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line 

Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used). Calculate the difference between 8 and 5. 	Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate. 	Find the difference between 8 and 5. 8 - 5, the difference is Children to explore why $9 - 6 = 8 - 5 = 7 - 4$ have the same difference.
Making 10 using ten frames. 14 - 5 	Children to present the ten frame pictorially and discuss what they did to make 10. 	Children to show how they can make 10 by partitioning the subtrahend. $14 - 5 = 9$  14 - 4 = 10 10 - 1 = 9

Introduced in Year 2

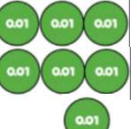
Concrete	Pictorial	Abstract
Column method using base 10. 48-7 	Children to represent the base 10 pictorially. 	$48 - 7 = 41$

Introduced in Year 3; consolidated in Year 4

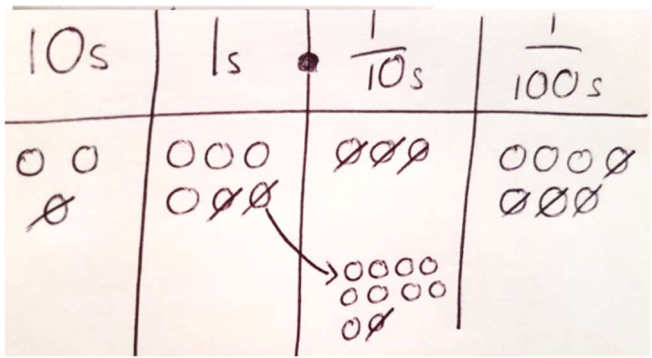
Concrete	Pictorial	Abstract
Column method using base 10. 48-7 	Children to represent the base 10 pictorially. 	Column method or children could count back 7. 
Column method using base 10 and having to exchange. 41 - 26 	Represent the base 10 pictorially, remembering to show the exchange. 	Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because $41 = 30 + 11$. 
Column method using place value counters. 234 - 88 	Represent the place value counters pictorially; remembering to show what has been exchanged. 	Formal column method. Children must understand what has happened when they have crossed out digits. 

Introduced in Year 5; consolidated in Year 6

Use of place value counters to subtract decimals.

Tens	Ones	Tenths	Hundredths
			

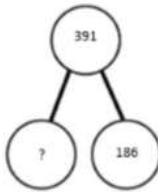
Children to represent the counters in a place value chart, circling when they make an exchange.



$$\begin{array}{r}
 39.137 \\
 - 11.44 \\
 \hline
 27.697
 \end{array}$$

Across year groups – different presentation of problems.

Conceptual variation; different ways to ask children to solve 391 - 186

 391 186 ?	Raj spent £391, Timmy spent £186. How much more did Raj spend? Calculate the difference between 391 and 186.	= 391 - 186 $\begin{array}{r} 391 \\ - 186 \\ \hline \end{array}$ What is 186 less than 391?	Missing digit calculations $\begin{array}{r} 39\Box \\ - \Box\Box6 \\ \hline \Box05 \end{array}$
---	--	---	--

Multiplication

Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

Year 1

Recognising and making equal groups, including doubling and counting in multiples - use cubes, Numicon and other objects in the classroom.

Year 2

Arrays- showing commutative multiplication

Year 3

Developing year 2 strategy: arrays

$2d \times 1d$ using base 10

Year 4

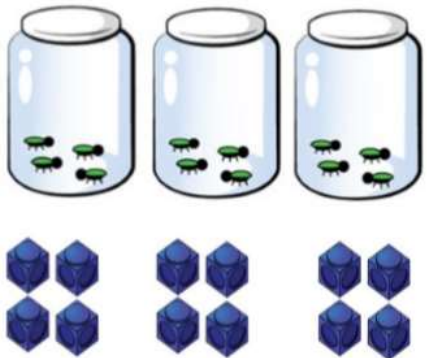
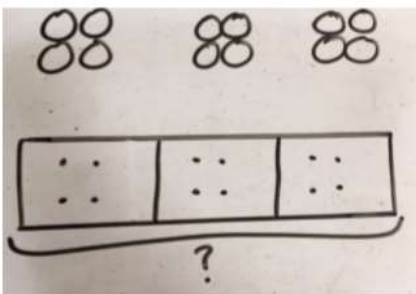
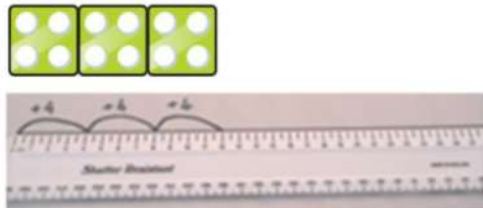
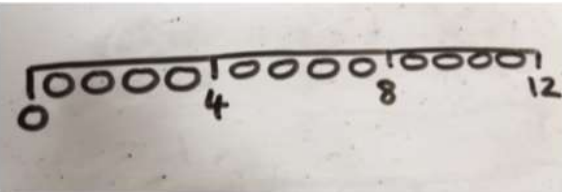
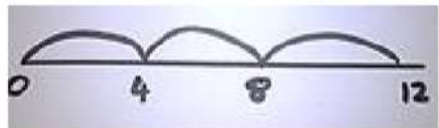
Column multiplication- introduced with place value counters (2 and 3 digit multiplied by 1 digit).

Year 5

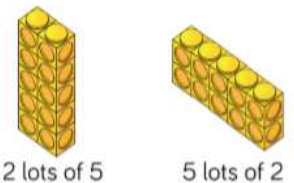
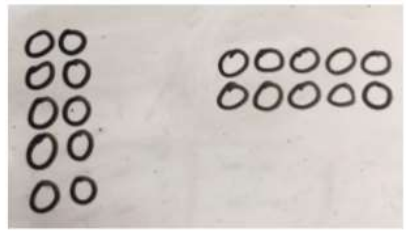
Column multiplication - abstract only but might need a repeat of year 4 first (up to 4 digit numbers multiplied by 1 or 2 digits)

Year 6

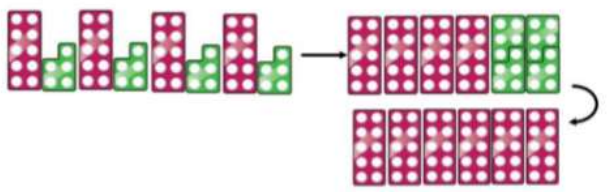
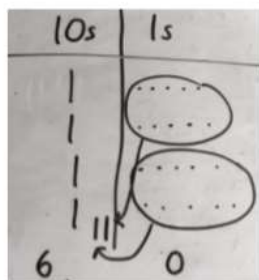
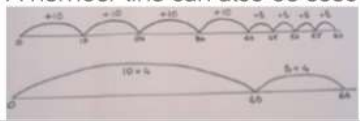
Developing year 5 strategy: Column multiplication – abstract methods (multi-digit up to 4 digits by a 2 digit number)

Concrete	Pictorial	Abstract
Repeated grouping/repeated addition 3×4 $4 + 4 + 4$ There are 3 equal groups, with 4 in each group. 	Children to represent the practical resources in a picture and use a bar model. 	$3 \times 4 = 12$ $4 + 4 + 4 = 12$
Number lines to show repeated groups- 3×4  Cuisenaire rods can be used too.	Represent this pictorially alongside a number line e.g.: 	Abstract number line showing three jumps of four. $3 \times 4 = 12$ 

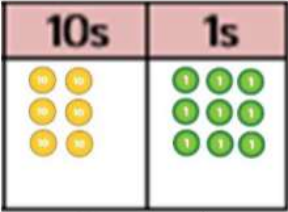
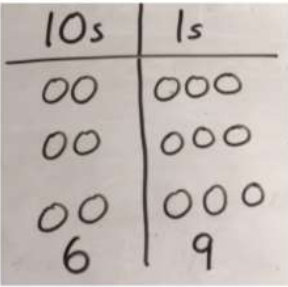
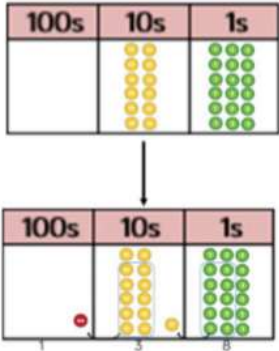
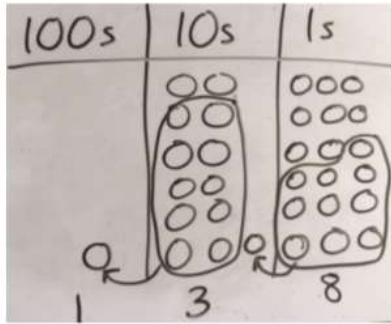
Introduced in Year 2

Concrete	Pictorial	Abstract
Use arrays to illustrate commutativity counters and other objects can also be used. $2 \times 5 = 5 \times 2$  2 lots of 5 5 lots of 2	Children to represent the arrays pictorially. 	Children to be able to use an array to write a range of calculations e.g. $10 = 2 \times 5$ $5 \times 2 = 10$ $2 + 2 + 2 + 2 + 2 = 10$ $10 = 5 + 5$

Introduced in Year 3

Concrete	Pictorial	Abstract
Partition to multiply using Numicon, base 10 or Cuisenaire rods. 4×15 	Children to represent the concrete manipulatives pictorially. 	Children to be encouraged to show the steps they have taken. 4×15 $10 \quad 5$ $10 \times 4 = 40$ $5 \times 4 = 20$ $40 + 20 = 60$ A number line can also be used 

Introduced in Year 4

Concrete	Pictorial	Abstract
Formal column method with place value counters (base 10 can also be used.) 3×23  6 9	Children to represent the counters pictorially. 	Children to record what it is they are doing to show understanding. $ \begin{array}{r} 3 \times 23 \\ \swarrow \searrow \\ 20 \quad 3 \end{array} $ $ \begin{array}{rcl} 3 \times 20 & = & 60 \\ 3 \times 3 & = & 9 \\ 60 + 9 & = & 69 \end{array} $ $ \begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array} $
Formal column method with place value counters. 6×23  1 3 8	Children to represent the counters/base 10, pictorially e.g. the image below. 	Formal written method $ \begin{array}{r} 6 \times 23 = \\ 23 \\ \times 6 \\ \hline 138 \\ \hline 1 \quad 1 \end{array} $

Introduced in Year 5; consolidated in year 6

Concrete	Pictorial	Abstract
When children start to multiply $3d \times 3d$ and $4d \times 2d$ etc., they should be confident with the abstract: To get 744 children have solved 6×124. To get 2480 they have solved 20×124.		$ \begin{array}{r} 124 \\ \times 26 \\ \hline 744 \\ 2480 \\ \hline 3224 \end{array} $ Answer: 3224

Across year groups – different presentation of problems.

Conceptual variation; different ways to ask children to solve 6×23																					
<table border="1"><tr><td>23</td><td>23</td><td>23</td><td>23</td><td>23</td><td>23</td></tr></table> ?	23	23	23	23	23	23	Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week? With the counters, prove that $6 \times 23 = 138$	Find the product of 6 and 23 $6 \times 23 =$ $= 6 \times 23$ <table><tr><td>6</td><td>23</td></tr><tr><td>$\times 23$</td><td>$\times 6$</td></tr><tr><td>—</td><td>—</td></tr></table>	6	23	$\times 23$	$\times 6$	—	—	What is the calculation? What is the product? <table border="1"><tr><th>100s</th><th>10s</th><th>1s</th></tr><tr><td></td><td> </td><td> </td></tr></table>	100s	10s	1s			
23	23	23	23	23	23																
6	23																				
$\times 23$	$\times 6$																				
—	—																				
100s	10s	1s																			

Division

Calculation policy: subtraction Key language: share, group, divide, divided by, half.

Year 1

Sharing objects into groups

Division as grouping e.g. I have 12 sweets and put them in groups of 3, how many groups? - use cubes and draw round 3 cubes at a time.

Year 2

Developing year 1 strategy: Division as grouping

Division within arrays- linking to multiplication

Repeated subtraction

Year 3

Division with a remainder-using lollipop sticks, times tables facts and repeated subtraction.

2d divided by 1d using base 10 or place value counters

Year 4

Developing year 3 strategy: Division with a remainder

Short division (up to 3 digits by 1 digit- concrete and pictorial)

Year 5

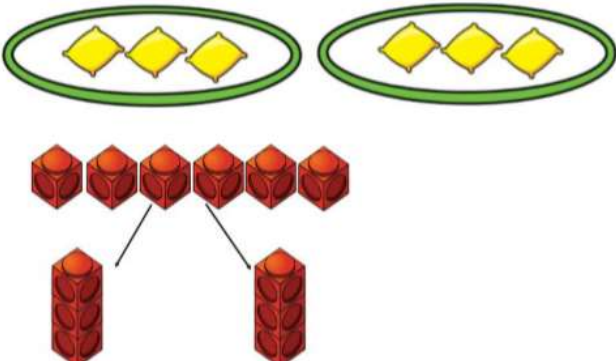
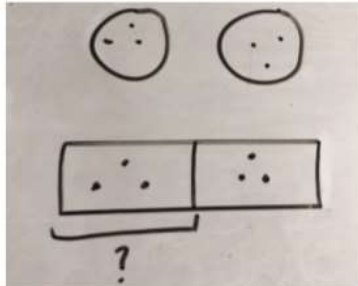
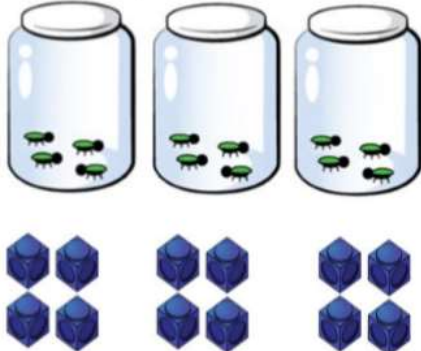
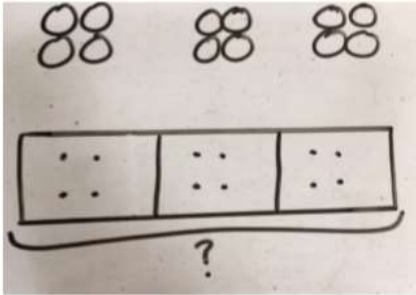
Short division (up to 4 digits by a 1 digit number including remainders- abstract)

Year 6

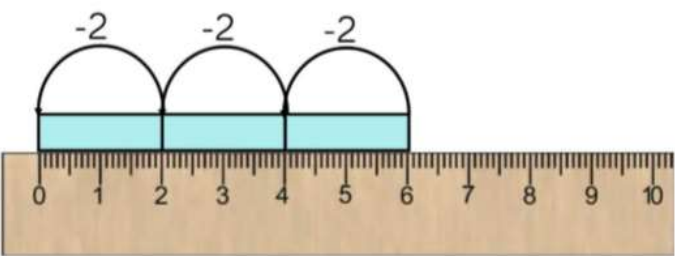
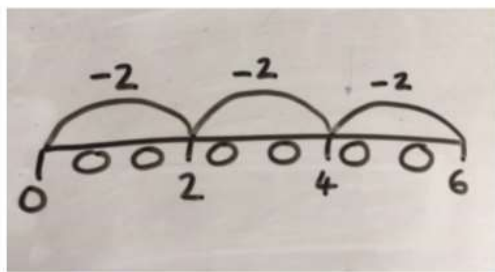
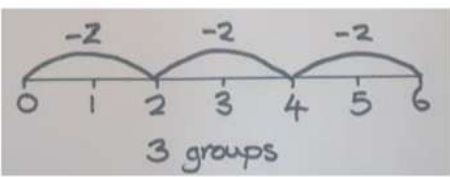
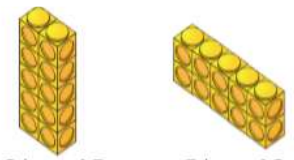
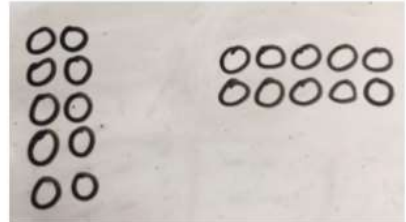
Developing year 5 strategy: Short division

Long division with place value counters (up to 4 digits by a 2 digit number) - children should exchange into the tenths and hundredths column too

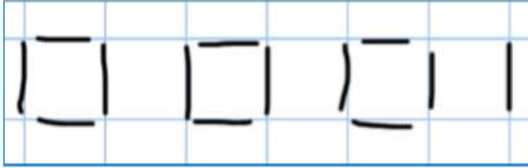
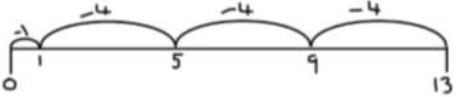
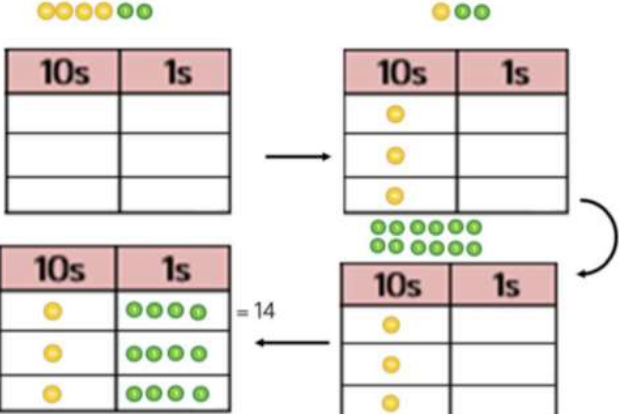
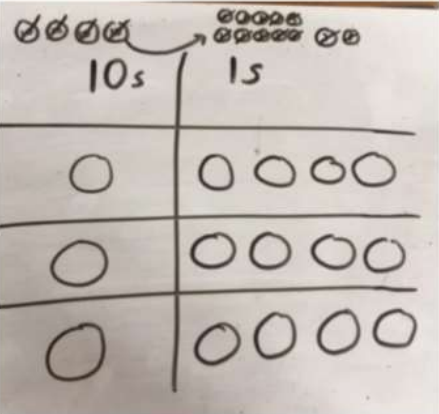
Introduced in Year 1

Concrete	Pictorial	Abstract		
Sharing using a range of objects. $6 \div 2$ 	Represent the sharing pictorially. 	$6 \div 2 = 3$ <table border="1" data-bbox="1516 354 1919 418"><tr><td>3</td><td>3</td></tr></table> Children should also be encouraged to use their 2 times tables facts.	3	3
3	3			
Division as grouping e.g. I have 12 and put it into groups of 4, how many groups do I have? Use of cubes or other objects, organising into groups of 4. 	Children to represent the practical resources in a picture and use a bar model. 	$12 = 4 + 4 + 4$ $12 \div 4 = 3$		

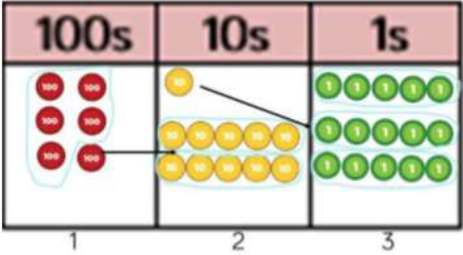
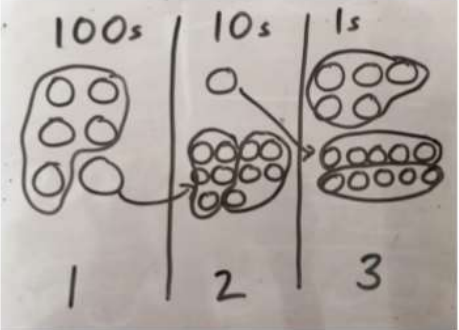
Introduced in Year 2

Concrete	Pictorial	Abstract
Repeated subtraction using Cuisenaire rods above a ruler. $6 \div 2$  3 groups of 2	Children to represent repeated subtraction pictorially. 	Abstract number line to represent the equal groups that have been subtracted. 
Use of arrays linked to multiplication counters or other objects can be used. $2 \times 5 = 10$ $5 \times 2 = 10$ $10 \div 5 = 2$ $10 \div 2 = 5$  2 lots of 5 5 lots of 2	Children to represent the arrays pictorially. 	Children to be able to use an array to write a range of calculations e.g. $2 \times 5 = 10$ $5 \times 2 = 10$ $10 \div 5 = 2$ $10 \div 2 = 5$

Introduced in Year 3

Concrete	Pictorial	Abstract
$2d + 1d$ with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used. $13 \div 4$ Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.  There are 3 whole squares, with 1 left over.	Children to represent the lollipop sticks pictorially.  There are 3 whole squares, with 1 left over.	$13 \div 4 = 3 \text{ remainder } 1$ Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line. '3 groups of 4, with 1 left over' 
Sharing using place value counters. $42 \div 3 = 14$ 	Children to represent the place value counters pictorially. 	Children to be able to make sense of the place value counters and write calculations to show the process. $42 \div 3$ $42 = 30 + 12$ $30 \div 3 = 10$ $12 \div 3 = 4$ $10 + 4 = 14$

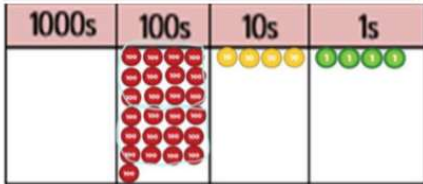
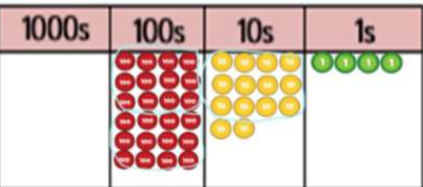
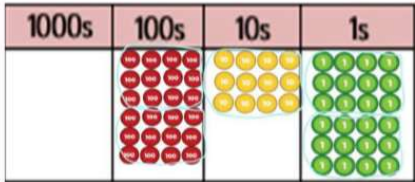
Introduced in Year 4

Concrete	Pictorial	Abstract
Short division using place value counters to group. $615 \div 5$  1. Make 615 with place value counters. 2. How many groups of 5 hundreds can you make with 6 hundred counters? 3. Exchange 1 hundred for 10 tens. 4. How many groups of 5 tens can you make with 11 ten counters? 5. Exchange 1 ten for 10 ones. 6. How many groups of 5 ones can you make with 15 ones?	Represent the place value counters pictorially. 	

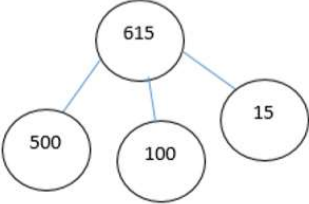
Introduced in Year 5

Concrete	Pictorial	Abstract
Concrete and pictorial methods introduced in year 4.		Children to the calculation using the short division scaffold. $\begin{array}{r} 123 \\ 5 \overline{) 615} \end{array}$

Introduced in Year 6

Concrete	Pictorial	Abstract
Long division using place value counters $2544 \div 12$		
	We can't group 2 thousands into groups of 12 so will exchange them.	
	We can group 24 hundreds into groups of 12 which leaves with 1 hundred.	$\begin{array}{r} 02 \\ 12 \overline{) 2544} \\ \underline{24} \\ 1 \end{array}$
	After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.	$\begin{array}{r} 021 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$
	After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.	$\begin{array}{r} 0212 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$

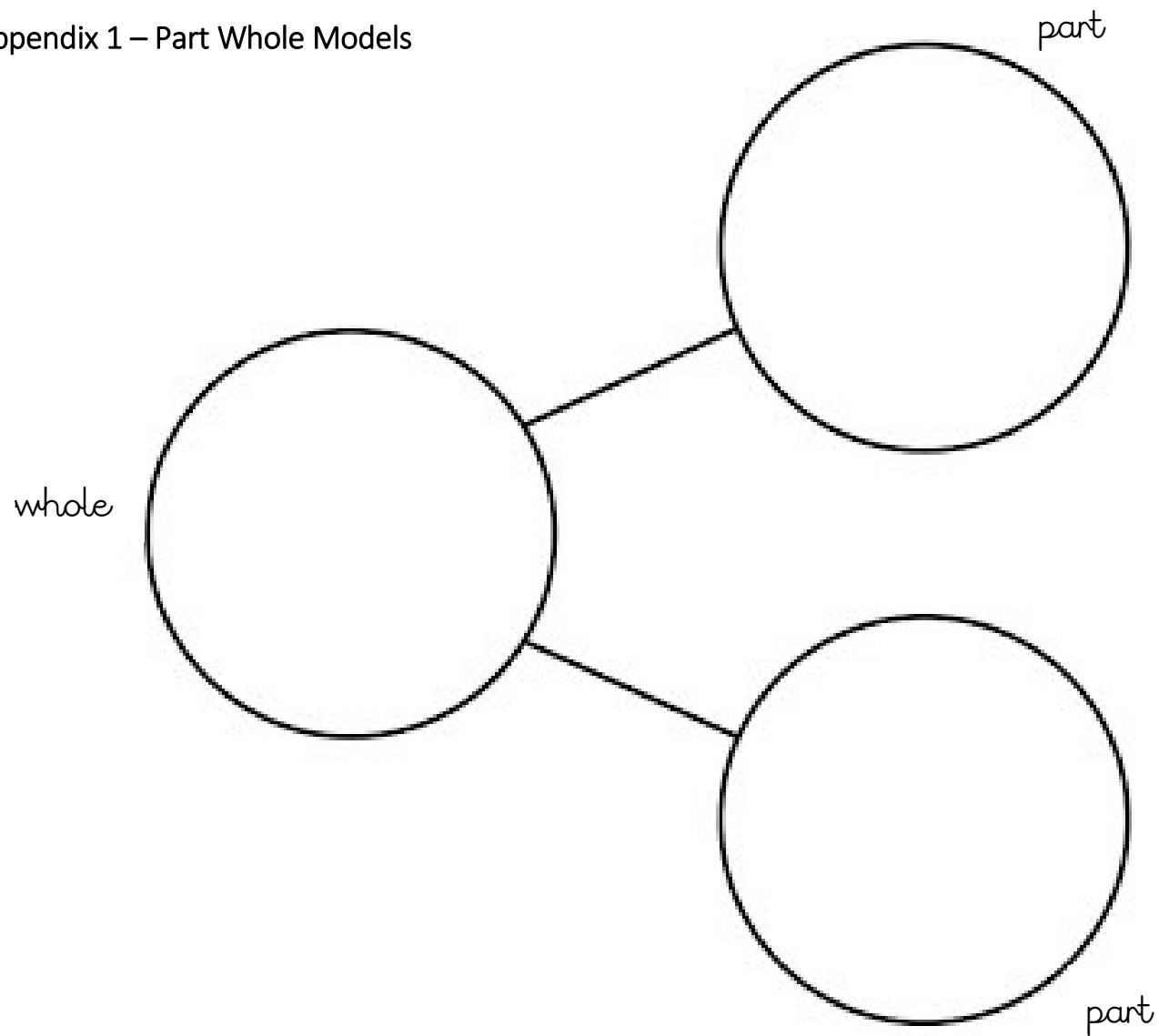
Across year groups – different presentation of problems.

Conceptual variation; different ways to ask children to solve $615 \div 5$			
Using the part whole model below, how can you divide 615 by 5 without using short division? 	I have £615 and share it equally between 5 bank accounts. How much will be in each account? 615 pupils need to be put into 5 groups. How many will be in each group?	$5 \overline{)615}$ $615 \div 5 =$ $\square = 615 \div 5$	What is the calculation? What is the answer? 

Appendices

These appendices provide some useful resources for presenting calculations in different ways and supporting children in calculating.

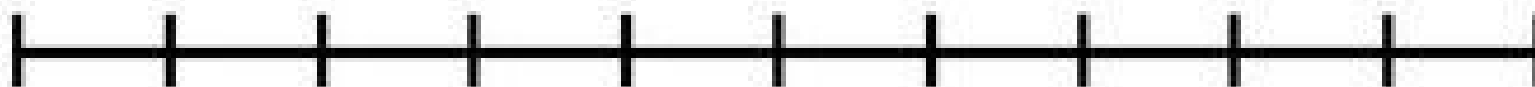
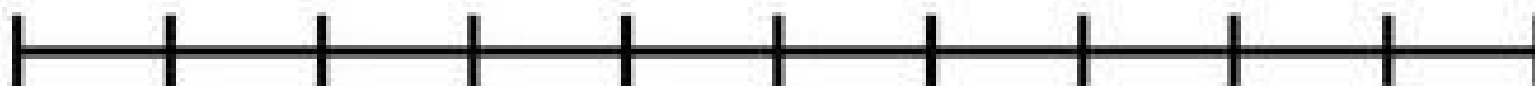
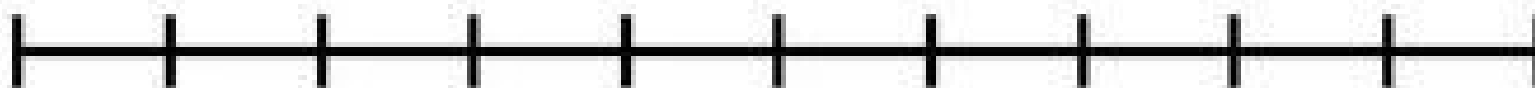
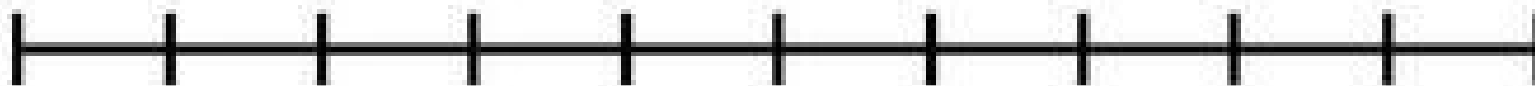
Appendix 1 – Part Whole Models



Whole	
Part	Part

Part	Part
Whole	

Appendix 2 – Blank Number Lines



Appendix 3 – Place Value Chart Year 2

Hundreds - 100s	Tens – 10s	Ones - 1s

Appendix 4 – Place Value Chart Year 3

Thousands – 1000s	Hundreds - 100s	Tens – 10s	Ones - 1s

Appendix 5 – Place Value Chart Year 4

Ten Thousands 10,000s	Thousands 1000s	Hundreds 100s	Tens 10s	Ones 1s	.	Tenths 1/10s 0.1s

Appendix 6 – Place Value Chart Year 5

Hundred Thousands 100,000s	Ten Thousands 10,000s	Thousands 1000s	Hundreds 100s	Tens 10s	Ones 1s	.	Tenths 1/10s 0.1s	Hundredths 1/100s 0.01s

Appendix 7 – Place Value Chart Year 6

Millions 1,000,000s	Hundred Thousands 100,000s	Ten Thousands 10,000s	Thousands 1000s	Hundreds 100s	Tens 10s	Ones 1s	.	Tenths 1/10s 0.1s	Hundredths 1/100s 0.01s	Thousandths 1/1000s 0.001s

Appendix 8 – Ten Frame
